SEMINARIO DEPARTAMENTO de MATEMÁTICA
UNIVERSIDAD TÉCNICA FEDERICO SANTA MARíA
Jueves 8 de Septiembre de 2016, 11:30
Sala de Seminarios, Edificio F, UTFSM

Sharp threshold for the van der Waerden's property

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This talk focuses on a popular line of research in extremal and probabilistic combinatorics in recent years. Roughly speaking it concerns problems that arise when one takes a notable combinatorial theorem and tries to prove that the same theorem holds in a sparse random subset of the ground set.

As an example, consider van der Waerden's theorem which states that every 2-coloring of the first n integers contains an arithmetic progression of length k, provided n is large enough depending on k. Besides being a classical result with a long and rich history this theorem remains influential up to date.

In the random version we consider $[n]_p$, the random subset of the first n integers obtained by picking each element with probability p independently. Rodl and Rucinski showed that there is a change of behavior around $q = n^{-1/(k-1)}$: If $p >> q$ then, with high probability, the random set $[n]_p$ satisfies the conclusion of van der Waerden's theorem whereas the conclusion fails with high probability if $p << q$.

With Friedgut, Person, and Schacht we improve this result in the setting of Z/nZ and show that the change in the behavior occurs very abruptly, namely already within a multiplicative constant of $(1 + o(1))$. This establishes what is call a sharp threshold!

The main purpose of the talk is to give an introduction into several questions and recent developments in extremal and probabilistic combinatorics and additive number theory. No prior experience required!